Chapter 3

Current Electricity

The branch of Physics which deals with the study of motion of electric charges is called **current electricity**.

In an uncharged metallic conductor at rest, some electrons are moving randomly through the conductor , as they are very loosely attached to the nuclei. The thermodynamic internal energy of the material is sufficient to liberate the outer electrons from individual atoms .It enables the electrons to travel through the material.
But the net flow of charge at any point in a particular direction is zero. Hence, there is **no flow of current.**
These electrons are termed as free electrons.
The external energy necessary to drive the free electrons in a definite direction is called **electromotive force (emf)**.
The emf is not a force, but it is the work done in moving a unit charge from one end to the other of a conductor.
The flow of free electrons in a conductor
constitutes electric current.

Electric current

Electric current is defined as the rate of flow of charges across any cross sectional area of a conductor.
If a net charge q passes through
any cross section of a conductor in time t, then the current
$I = q / t$, where q is in coulomb and t is in second. The current I is expressed in ampere.
If the rate of flow of charge is not uniform, the currentvaries with time and the instantaneous value of current i is given by,

$$i = dq/dt$$

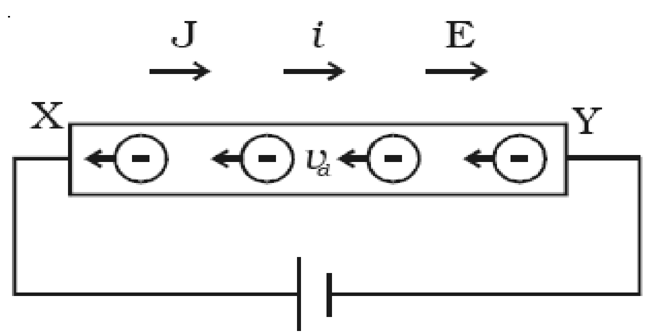
Current is a **scalar quantity**. The direction of conventional current is taken as the direction of flow of positive charges or **opposit**e to the direction of flow of electrons.

Drift velocity and mobility

Consider a conductor XY connected to a battery as in the figure. A steady electric field **E** is established in the conductor in the direction X to Y.

In the absence of an electric field, the free electrons in the conductor move randomly in all possible directions so the average thermal velocity u is zero.

They do not constitute electric current.



But, as soon as an electric field **E** is applied, the free electrons at the end Y experience a force **F = eE** in a direction opposite to the electric field.

The electrons are accelerated and in the process they collide with each other and with the positive ions in the conductor.

Thus due to collisions, a backward force acts on the electrons and they are slowly drifted with a constant average drift velocity $\boldsymbol{v_d}$ in a direction opposite to electric field.

Drift velocity is defined as the average velocity with which free electrons get drifted towards the positive terminal, when an electric field is applied.

τ is the average time between two successive collisions it is known as **relaxation time**.
If the acceleration experienced by the electron be *a*, then the drift velocity isgiven by,
$\boldsymbol{v_d = a\tau}$

The force experienced by the electron of mass *m* is
$\mathbf{F} = m\mathbf{a}$

$$\text{Hence} \quad a = \frac{eE}{m}$$

$$\therefore v_d = \frac{eE}{m}\tau = \mu E$$

The drift velocity of electrons is proportional to the electric field intensity.
It is very small and is of the order of $0.1\ cm\ s^{-1}$.

Where μ is the mobility and is defined as the drift velocity acquired per unit electric field.
Its SI unit $m^2V^{-1}s^{-1}$.

$$\mu = v_d / E = e\tau / m$$

Mobility increases with increase in drift velocity as temperature increases τ decreases so v_d and μ decreases.

Even though v_d is proportional to the electric field intensity E, μ is independent of E.

Current density

Current density at a point is defined as the quantity of charge passing per unit time through unit area, taken perpendicular to the direction of flow of charge at that point.

The current density **J** for a current I flowing across a conductor having an area of cross section A is

$$\mathbf{J} = \frac{(q/t)}{A} = \frac{I}{A}$$

Current density is a vector quantity. It is expressed in A m^{-2}.

Relation between current and drift velocity

Consider a conductor XY of length L and area of cross section A. An electric field E is applied between its ends.

Let n be the number of free electrons per unit volume.

The free electrons move towards the left with a constant drift velocity vd.

The number of conduction electrons in the conductor = nAL

The charge of an electron = e

The total charge passing through the conductor q = (nAL) e

The time in which the charges pass through the conductor

$$t = \frac{L}{v_d}$$

The current flowing through the conductor,

$$I = \frac{q}{t} = \frac{(nAL)e}{(L/v_d)}$$

$$I = nAev_d \qquad ...(1)$$

The current flowing through a conductor is directly proportional to the drift velocity.

From equation (1),

$$\frac{I}{A} = nev_d \qquad J = nev_d \qquad \left[\because J = \frac{I}{A}, \text{current density}\right]$$

1 .If 6.25 × 10^{18} electrons flow through a given cross section in unit time, find the current. (Given: Charge of an electron is 1.6 × 10^{-19}C)

n = 6.25 × 10^{18} ; e = 1.6 × 10^{-19} C ;t = 1 s ; I = ?

$$I = \frac{q}{t} = \frac{ne}{t} = \frac{6.25\times10^{18}\times1.6\times10^{-19}}{1} = 1 \text{ A}$$

2.***A copper wire of 10^{-6} m^2 area of cross section, carries a current of 2 A. If the number of electrons per cubic metre is 8 × 10^{28},calculate the current density and average drift velocity.(Given e = 1.6 × 10^{-19}C)***

A = 10^{-6}m^2; Current flowing I = 2 A n =8×10^{28}/m^3
e = 1.6 × 10^{-19} C ; J = ? ; v d =?

$$\text{Current density, } J = \frac{I}{A} = \frac{2}{10^{-6}} = 2 \times 10^6 \text{A/m}^2$$

$$J = n e v_d$$

$$v_d = \frac{J}{ne} = \frac{2\times10^6}{8\times10^{28}\times1.6\times10^{-19}} = 15.6 \times 10^{-5}\ \mathrm{m\,s^{-1}}$$

Ohm's law

George Simon Ohm established the relationship between potential difference and current, which is known as Ohm's law. The steady current flowing through a conductor is directly proportional to the potential difference between the ends of the conductor ,when the physical conditions like temperature remain constant .

The current flowing through a conductor is,

$$I = nAev_d$$

$$\text{But} \quad v_d = \frac{eE}{m} \cdot \tau$$

$$\therefore \quad I = nAe\frac{eE}{m}\tau$$

$$I = \frac{nAe^2}{mL}\tau V \qquad \left[\because E = \frac{V}{L}\right]$$

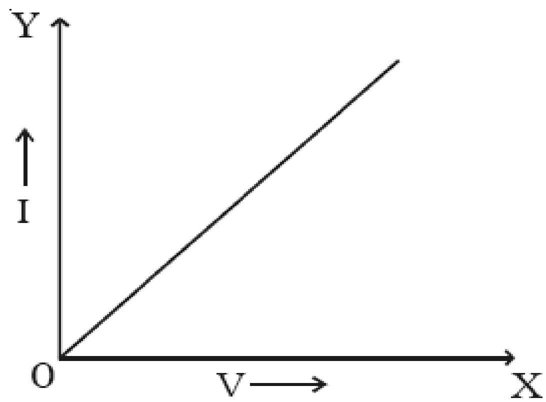
where V is the potential difference. The quantity $\frac{mL}{nAe^2\tau}$ is a constant for a given conductor, called electrical resistance (R).

$$\text{(i.e)} \quad I \propto V \quad \text{or} \quad I$$

$$\therefore \quad V = IR \quad \text{or} \quad R$$

Resistance R of a conductor is defined as the ratio of potential difference across the conductor to the current flowing through it. It is the opposition to the flow of electric current given by the conductor.
The unit of resistance is **ohm (Ω**)
The reciprocal of resistance is
Conductance . Its unit is **mho (Ω^{-1}). Or siemen**

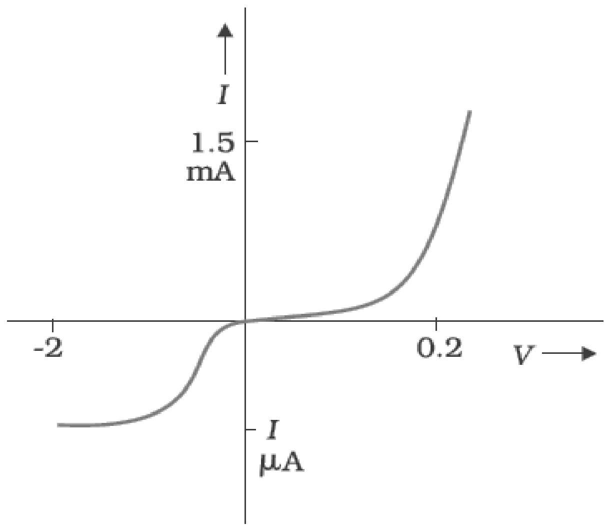
The graph between V and I is a straight line.



The slope of the graph gives the conductance and the reciprocal of the slope is resistance.

LIMITATIONS OF OHM'S LAW
Although Ohm's law has been found valid over a large class of materials, there do exist materials and devices used in electric circuits where the proportionality of V and I does not hold.
The deviations broadly are one or more of the following types:
(a) V ceases to be proportional to I

(b) The relation between V and I depends on the sign of V. In other words, if I is the current for a certain V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction.



The figure above is the I-V characteristic curve of a Diode.

(c) The relation between V and I is not unique, i.e., there is more than one value of V for the same current I.

A material exhibiting such behavior is Ga As.

The devices like **Diode ,Transistor** for which Ohm's Law does not hold good are Known as **non Ohmic devices.**

Resistivity :

In general, resistance R is proportional to

length, *R α l .... (1)*

and the resistance R is inversely proportional to the cross-sectional area A

$$R \propto \frac{1}{A} \quad \textbf{(2)}$$

combining equations (1) and (2) we have

$$R \propto \frac{l}{A}$$

and hence for a given conductor

$$R = \rho \ \frac{l}{A}$$

where the constant of proportionality ρ depends on the material of the conductor but not on its dimensions.
ρ is called resistivity.
Using the last equation, Ohm's law reads

$$V = I \times R = \frac{I\rho l}{A}$$

Relation between E, j , ρ and σ

Current per unit area (taken normal to the current), I/A, is called current density and is denoted by j. The SI units of the current density are A/m^2. Further, if E is the magnitude of uniform electric field in the
conductor whose length is l, then the potential difference V across it sends is El. Using these, the last equation reads

$$\mathbf{V} = \mathbf{E}\, l = \mathbf{I}\boldsymbol{\rho}\, \mathbf{l}/\mathbf{A} = \mathbf{j}\, \rho\, l$$

$$\textbf{or,}\ \mathbf{E} = \mathbf{j}\, \rho$$

The above relation for magnitudes E and j can indeed be cast in a vector form. The current density, (which we have defined as the current through unit area normal to the current) is also directed along **E**, and is also a vector **j** Thus, the last equation can be written as,

$$\mathbf{E} = \mathbf{j}\rho \qquad \mathbf{j} = \mathbf{E}/\boldsymbol{\rho}$$

$$\text{or, } \mathbf{j} = \sigma\mathbf{E}$$

where $\boldsymbol{\sigma} \equiv \mathbf{1}/\boldsymbol{\rho}$ is called the conductivity.

RESISTIVITY OF VARIOUS MATERIALS

The resistivities of various common materials are listed in Table.

The materials are classified as conductors, semiconductors and insulators depending on their resistivities , in an increasing order of their values.

Metals have low resistivities in the range of $10^{-8}\ \Omega$ m to $10^{-6}\Omega$m.

At the other end are insulators like ceramic, rubber and plastics having resistivities 10^{18} times greater than metals or more.

In between the two are the semiconductors. These, however, have resistivities characteristically decreasing with a rise in temperature.

The resistivities of semiconductors are also affected by presence of small amount of impurities. This last feature is exploited in use of semiconductors for electronic devices.

Class assignment

1.What is the effect of increase in temeperature on i) resistivity ii) conductivity of a conductor? Justify

2***.Name the factors which affect the resistivity of semiconductors***.

3. ***Even though copper has less resistivity than Aluminium, Aluminiumis used for transmission of power over long distances why?***

4. Why alloys like Nichrome are used as heating elements of heating devices?

Material	Resistivity, ρ (Ω m) at 0°C	Temperature coefficient of resistivity, α (°C)$^{-1}$ $\frac{1}{\rho}\left(\frac{d\rho}{dT}\right)$ at 0°C
Conductors		
Silver	1.6×10^{-8}	0.0041
Copper	1.7×10^{-8}	0.0068
Aluminium	2.7×10^{-8}	0.0043
Tungsten	5.6×10^{-8}	0.0045
Iron	10×10^{-8}	0.0065
Platinum	11×10^{-8}	0.0039
Mercury	98×10^{-8}	0.0009
Nichrome (alloy of Ni, Fe, Cr)	$\sim 100 \times 10^{-8}$	0.0004
Manganin (alloy)	48×10^{-8}	0.002×10^{-3}
Semiconductors		
Carbon (graphite)	3.5×10^{-5}	- 0.0005
Germanium	0.46	- 0.05
Silicon	2300	- 0.07
Insulators		
Pure Water	2.5×10^{5}	
Glass	$10^{10} - 10^{14}$	
Hard Rubber	$10^{13} - 10^{16}$	
NaCl	$\sim 10^{14}$	
Fused Quartz	$\sim 10^{16}$	

Commercially produced resistors for domestic use or in laboratories are of two major types: **wire bound resistors and carbon resistors.**

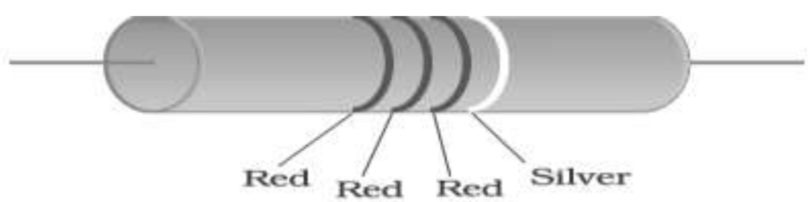
Wire bound resistors are made by winding the wires of an alloy, viz., manganin,

constantan, nichrome or similar ones. The choice of these materials is dictated mostly by the fact that their resistivities are relatively insensitive to temperature.
These resistances are typically in the range of a fraction of an ohm to a few hundred ohms.
Resistors in the higher range are made mostly from carbon.
Carbon resistors are compact, inexpensive and thus find extensive use in electronic circuits.
Carbon resistors are small in size and hence their values aregiven using a colour code.
Colour Code

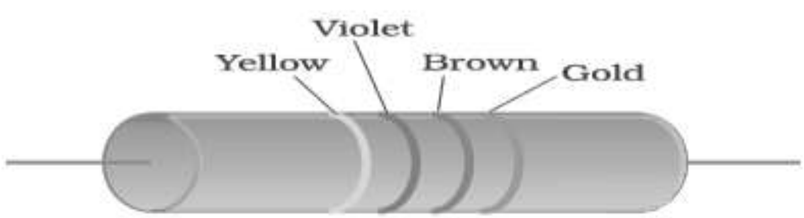
Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

The resistors have a set of co-axial coloured rings on them whose significance are listed in Table.
The **first two bands** from the end indicate the first two **significant figures** of the resistance in ohms.
The **third band** indicates the **decimal multiplier** (as listed in Table).
The **last band** stands for **toleranc**e or possible variation in percentage about the indicated values.
Sometimes, this last band is absent and that indicates a tolerance of 20% .
For example, if the four colours are **orange, blue, yellow and gold**, the resistance value is **$36 \times 10^4\ \Omega$,** with a tolerence value of **5%.**

Text book questions
1.*With reference to the table given find the resistance of the carbon resistors in the given figure a and b.*



(a)



(b)

Solution:
In figure (a) $22 \times 10^2 \Omega$, with a tolerance of 10%.
In figure (b) $47 \times 10^1\ \Omega$, with a tolerance of 5%.

2. ***a) The electron drift speed is estimated to be only a few mm s^{-1} for currents in the range of a few amperes? How then is current established almost the instant a circuit is closed?***

(b) The electron drift arises due to the force experienced by electrons in the electric field inside the conductor. But force should cause acceleration. Why then do the electrons acquire a steady average drift speed?

(c) If the electron drift speed is so small, and the electron's charge is small, how can we still obtain large amounts of current in a conductor?
(d) When electrons drift in a metal from lower to higher potential, does it mean that all the 'free' electrons of the metal are moving in the same direction?
(e) Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the (i) absence of electric field, (ii) presence of electric field?

Solution:
(a) Electric field is established throughout the circuit, almost instantly (with the speed of light) causing at every point a local electron drift. Establishment of a current does not have to wait for electrons from one end of the conductor travelling to the other end. However, it does take a little while for the current to reach its steady value.
(b) Each 'free' electron does accelerate, increasing its drift speed until it collides with a positive ion of the metal. It loses its drift speed after collision but starts to accelerate and increases its

drift speed again only to suffer a collision again and so on. On the average, therefore, electrons acquire only a drift speed.
(c) Simple, because the electron number density is enormous,~10^{29} m^{-3}.
(d) By no means. The drift velocity is superposed over the large random velocities of electrons.
(e) In the absence of electric field, the paths are straight lines; in the presence of electric field, the paths are, in general, curved**.**

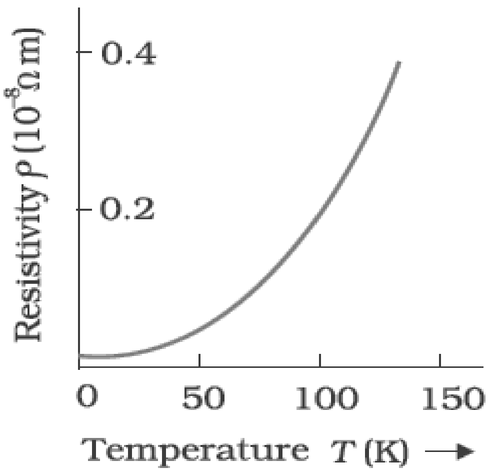
TEMPERATURE DEPENDENCE OF RESISTIVITY

The resistivity of a material is found to be dependent on the temperature.
Different materials do not exhibit the same dependence on temperatures.
Over a limited range of temperatures, that is not too large, the resistivity of a metallic conductor is approximately given by,

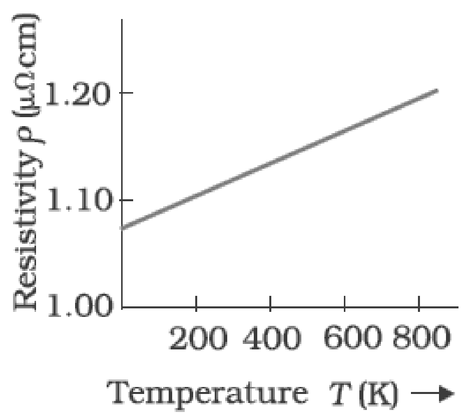
$$\rho_T = \rho_0 [1 + \alpha (T - T_0)]$$

where ρ_T is the resistivity at a temperature T and ρ_0 is the same at a reference temperature T_0. α is called the temperature co-efficient of resistivity, the dimension of α is $(\text{Temperature})^{-1}$ and unit is K^{-1}.

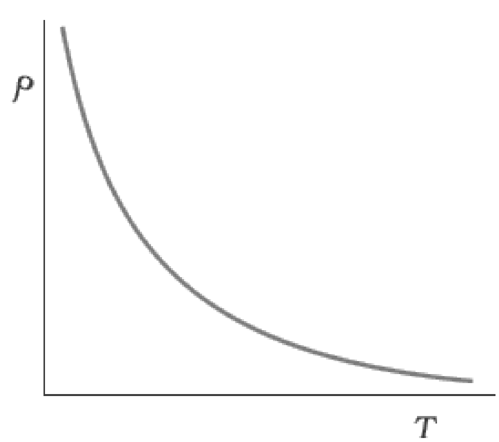
For metals, α is high positive, alloys low positive and for insulators it is negative.



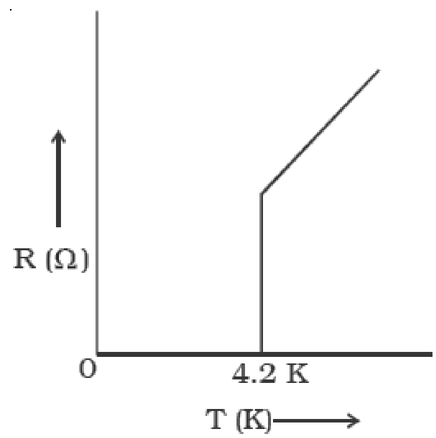
Variation of ρ with temperature for Copper a metal.



Variation of ρ with temperature for Nichrome an alloy .



Variation of ρ with temperature for Silicon a semiconductor.



Variation of ρ with temperature for a superconductor(mercury at 4.2K will become a super conductor) .

Some **materials like Nichrome** (which is an alloy of nickel, iron and chromium) , **Manganin and constantan** exhibit a very weak dependence of resistivity with temperature so they are used to make standard resistance coils.
We can qualitatively understand the temperature dependence of resistivity,

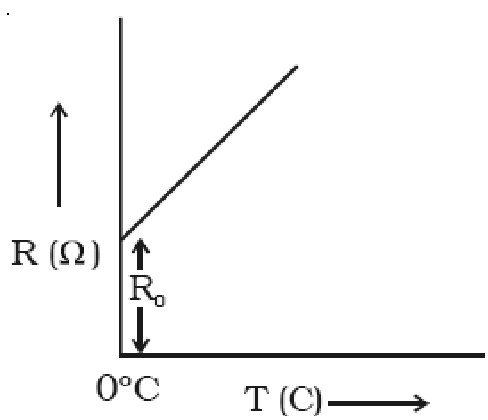
$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

ρ thus depends inversely both on the number n of free electrons per unit volume and on the average time τ between collisions.
As we increase temperature, average speed of the electrons, which act as the carriers of

current, increases resulting in more frequent collisions. The average time of collisions τ, thus decreases with temperature , so the resistivity of a metal increases with increase in temperature. In a metal, n is not dependent on temperature to any appreciable extent.
For **insulators and semiconductors**, however, n increases with temperature. This increase more than compensates any decrease in τ in so that for such materials, ρ decreases with temperature.

Temperature dependence of resistance
For **conductors** the resistance increases with increase in temperature. If R_o
is the resistance of a conductor at 0o C and R_t is the resistance of same conductor at to C, then
$\mathbf{R_t = R_o (1 + \alpha t)}$
where α is called the temperature
coefficient of resistance.

$$\alpha = \frac{R_t - R_o}{R_o t}$$



The temperature coefficient of resistance is defined as fractional change in resistance per unit change in temperature.

R_t, R_0 are the resistances at t◦C and 0◦ C respectively.

In the graph **slope** gives change in resistance per unit change in temperature and the **Y intercept** gives resistance at 0^0C**.**

3. The resistance of a nichrome wire at 0^o C is 10 Ω. If its temperature coefficient of resistance is 0.004 / ^{o}C, find its resistance at boiling point of water.

,$R_o = 10\ \Omega$; $\alpha = 0.004/^oC$; $t = 100^0C$;

, $R_t = ?$

$R_t = R_o (1+ \alpha t)$

$= 10 (1 + (0.004 \times 100))$

$R_t = 14\ \Omega$

4.Two wires of same material and length have resistances 5 Ω and 10 Ω respectively. Find the ratio of radii of the two wires.

Resistance of first wire $R_1 = 5\ \Omega$;

Radius of first wire = r_1

Resistance of second wire $R_2 = 10\ \Omega$

Radius of second wire = r_2

Length of the wires = *l*

Specific resistance of the material of the wires = ρ

$$R = \frac{\rho l}{A}; A = \pi r^2$$

$$R_1 = \frac{\rho l}{\pi r_1^2} ; \; R_2 = \frac{\rho l}{\pi r_2^2}$$

$$\frac{R_2}{R_1} = \frac{r_1^2}{r_2^2} \quad \text{or} \quad \frac{r_1}{r_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{10}{5}} = \frac{\sqrt{2}}{1}$$

$$r_1 : r_2 = \sqrt{2} : 1$$

Combination of resistors

In simple circuits with resistors, Ohm's law can be applied to find the effective resistance. The resistors can be connected in series and parallel.

Resistors in series

Let us consider the resistors of resistances R_1,R_2, R_3 and R_4 connected in series as shown in Fig .When resistors are connected in series, the current flowing through each resistor is the same. If the potential difference applied between the ends of the combination of resistors is V, then the potential difference across each resistor R_1, R_2, R_3 and R_4 is V_1, V_2,V_3 and V_4 respectively.

The net potential difference $V = V_1 + V_2 + V_3 + V_4$

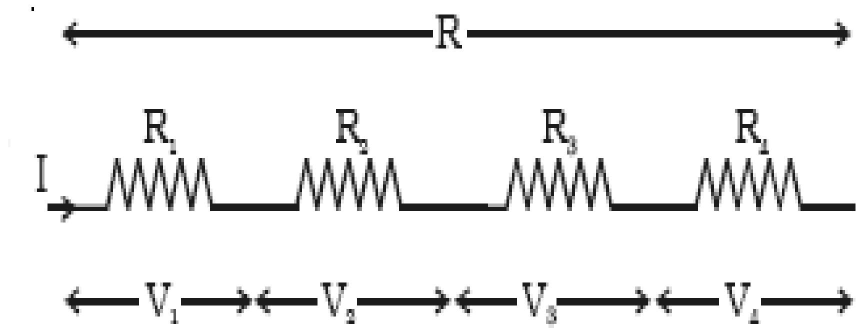
By Ohm's law

$V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$, $V_4 = IR_4$ and $V = IR_s$

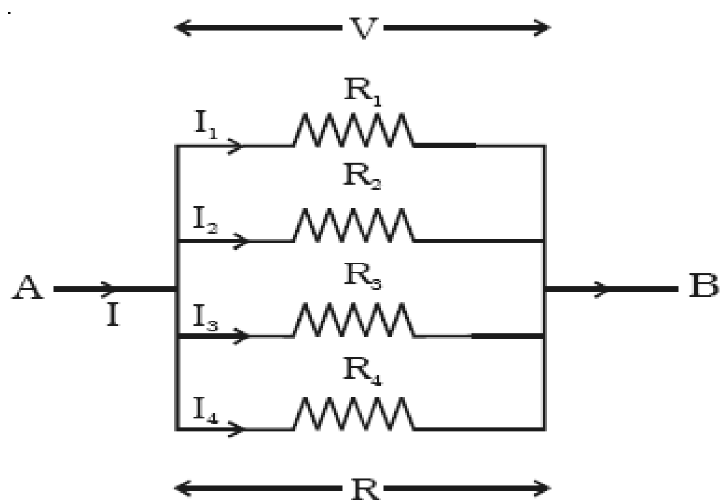
where R_S is the equivalent or effective resistance of the series combination.

Hence, $IR_S = IR_1 + IR_2 + IR_3 + IR_4$

$R_S = R_1 + R_2 + R_3 + R_4$
Thus, the equivalent resistance of a number of resistors in series connection is equal to the sum of the resistance of individual resistors.



Resistors in parallel
Consider four resistors of resistances R_1, R_2, R_3 and R_4 connected in parallel .



A source of emf V is connected to the parallel combination. When resistors are in parallel, the potential difference (V) across each resistor is the same.
A current I entering the combination gets divided into I_1, I_2, I_3 and I_4 through R_1, R_2, R_3 and R_4

respectively, such that

$$I = I_1 + I_2 + I_3 + I_4.$$

By Ohm's law

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ I_3 = \frac{V}{R_3},\ I_4 = \frac{V}{R_4} \text{ and } I = \frac{V}{R_P}$$

where R_P is the equivalent or effective resistance of the parallel combination.

$$\therefore \frac{V}{R_P} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the resistance of the individual resistors is equal to the reciprocal of the effective resistance of the combination.

Note:

If n identical resistances each of resistance R are connected in

i) series then effective resistance R_s = n R

ii) parallel then effective resistance R_p= R/n

Internal resistance of a cell

The electric current in an external circuit flows from the positive terminal to the negative terminal of the cell, through different circuit elements.

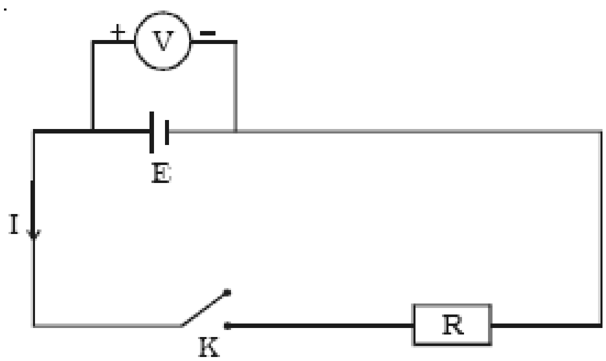
In order to maintain continuity, the current has to flow through the electrolyte of the cell, from its negative terminal to positive terminal.

During this process of flow of current inside the cell, a resistance is offered to current flow by the electrolyte of the cell. This is termed as the internal resistance of the cell.

A freshly prepared cell ha**s low internal resistance.**

Determination of internal resistance of a cell using voltmeter

The circuit connections are made as shown in Figure below .



EMF is defined as the potential difference between the terminals of the cell when no current is drawn or the circuit is open.

With key K open, the emf of cell E is found by connecting a high resistance voltmeter across it.

Since the high resistance voltmeter draws only a very feeble current for deflection, the circuit may be considered as an open circuit.

Hence the voltmeter reading gives the emf of the cell.
Terminal Potential difference is defined as the potential difference between the terminals of the cell when current is drawn from the cell or the circuit is closed.
A small value of resistance R is included in the external circuit and key K is closed. The potential difference across R is equal to the potential difference across cell (V).The potential drop across R, V = IR ...(1)
Due to internal resistance r of the cell, the voltmeter reads avalue V, less than the emf of cell.
Then V = E – Ir or Ir = E−V ...(2)
Dividing equation (2) by equation (1)

$$\frac{Ir}{IR} = \frac{E-V}{V} \quad \text{or} \quad r = \left(\frac{E-V}{V}\right)R$$

Since E, V and R are known, the internal resistance *r* of the cellcan be determined.
Factors affecting internal resistance of a cell
Internal resistance **r** increases with
i) Increase in distance between the electrodes
ii) decrease in temperature
iii) decrease in area of the electrode

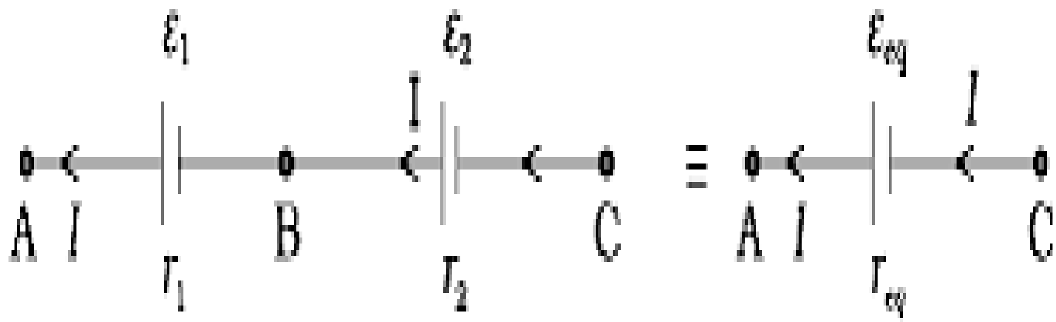
Can terminal Pd be more than emf of a secondary cell or a battery ?
V = E – Ir, from this equation we find
i)V < E when the circuit is closed or current is drawn from the cell .

ii) V =E ,when the circuit is open.
iii) V> E when the cell is charged by a dc supply.

Cells in Series & Parallel

Like resistors, cells can be combined together in an electric circuit. And
like resistors, one can, for calculating currents and voltages in a circuit,
replace a combination of cells by an equivalent cell.

Cells in series



Consider first two cells in series in the figure above , where one terminal of the two cells is joined together leaving the other terminal in either cell free.
ε1, ε2 are the emf's of the two cells and *r*1, *r*2 their internal resistances, respectively.

Let *V* (A), *V* (B), *V* (C) be the potentials at points A, B and C in the figure
Then *V* (A) – *V* (B) is the potential difference between the positive and negative terminals of the first cell.

$$V_{AB} \equiv V(A) - V(B) = \varepsilon_1 - I r_1$$

Similarly,

$$V_{BC} \equiv V(B) - V(C) = \varepsilon_2 - I r_2$$

The Pd between A and C is

$$V_{AC} \equiv V(A) - V(C) = \lfloor V(A) - V(B) \rfloor + \lfloor V(B) - V(C) \rfloor$$

$$= (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$$

The effective Pd due to a single cell of emfε_{eq} and internal resistance r_{eq}

$$V_{AC} = \varepsilon_{eq} - I r_{eq}$$

Comparing the last two equations, we get

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$$

$$\text{and } \; r_{eq} = r_1 + r_2$$

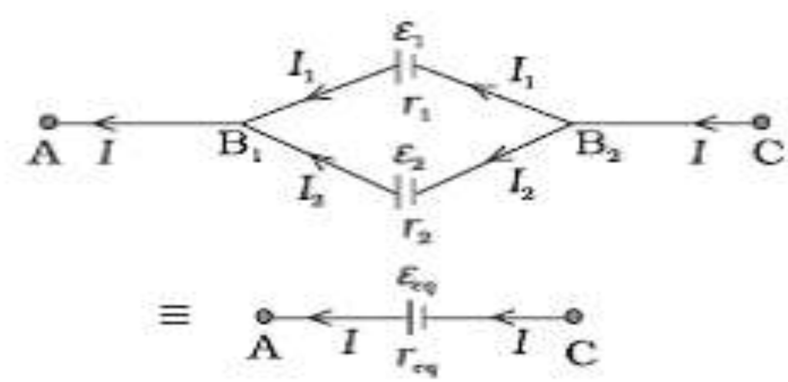
In the figure we had connected the negative electrode of the first to the positive electrode of the second. If instead we connect the two negatives, the above two equations would change to

$$\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2 \qquad (\varepsilon_1 > \varepsilon_2)$$

$r_{eq} = r_1 + r_2$

Note:
(i) The equivalent emf of a series combination of n cells is just the sum of their individual emf's, and
(ii) The equivalent internal resistance of a series combination of n cells is just the sum of their internal resistances.
This is so, when the current leaves each cell from the positive electrode.
If in the combination, the current leaves any cell from the *negative* electrode, the emf of the cell in which current enters has a **negative sign** ,in the expression for ε_{eq}.

Cells in parallel
Next, consider a parallel combination of the cells .



I_1 and I_2 are the currents leaving the positive electrodes of the cells. At the point B1, I_1 and I_2 flow in whereas the current I flows out. Since as much charge flows in as out, we have

$$I = I_1 + I_2$$

Let $V(B1)$ and $V(B2)$ be the potentials at $B1$ and $B2$, respectively.
Then, considering the first cell, the potential difference across its terminals is $V(B1) – V(B2)$.

$$V \equiv V(B_1) - V(B_2) = \varepsilon_1 - I_1 r_1$$

Points B_1 and B_2 are connected exactly similarly to the second cell. Hence considering the second cell, we also have

$$V \equiv V(B_1) - V(B_2) = \varepsilon_2 - I_2 r_2$$

Combining the last three equations

$$I = I_1 + I_2$$

$$= \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Hence, V is given by,

$$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\frac{r_1 r_2}{r_1 + r_2}$$

If we replace the combination by a single cell, between $B1$ and$B2$, of emf ε_{eq} and internal resistance r_{eq}, we would have

$$V = \varepsilon_{eq} - I\, r_{eq}$$

comparing the last two equations

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

in simple form

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

If there an n cells of emfε_{1}, . . .ε_{n} and of internal resistancesr_1, . . . r_n respectively, connected in parallel, the combination is equivalent to a single cell of emf$\varepsilon_{eq}$and internal resistance r_{eq}, such that

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \cdots + \frac{1}{r_n}$$

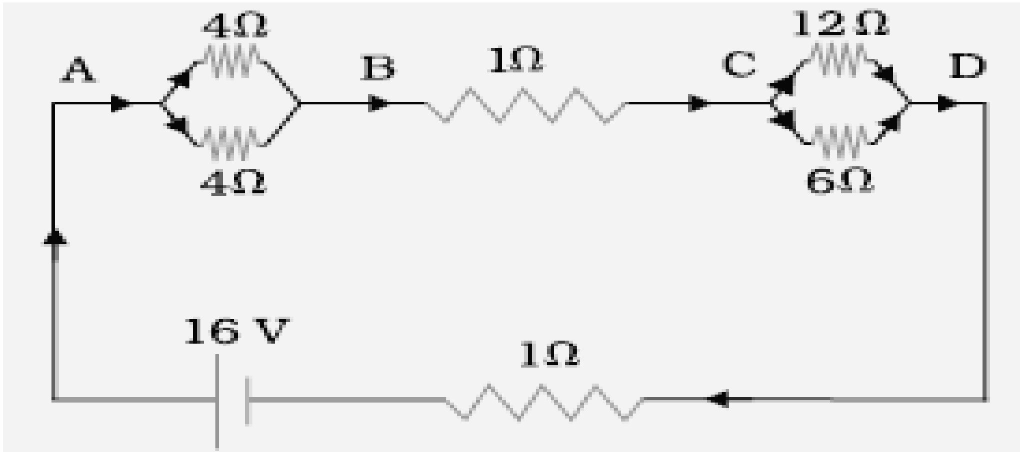
$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \cdots + \frac{\varepsilon_n}{r_n}$$

Class assignment:

Derive expression for effective emf of n identical cells each of emf ε connected in series and m rows of such combinations are connected in parallel. Get the expression for total current also.

Text book example:

1.A network of resistors is connected to a 16 V battery with internal resistance of 1Ω, as shown in Fig. (a) Compute the equivalent resistance of the network. (b) Obtain the current in each resistor. (c) Obtain the voltage drops VAB, VBC and VCD



(a) The network is a simple series and parallel combination ofresistors. First the two 4Ω resistors in parallel are equivalent to aresistor

$$= [(4 \times 4)/(4 + 4)]\ \Omega = 2\ \Omega.$$

In the same way, the 12 Ω and 6 Ω resistors in parallel areequivalent to a resistor of

$$[(12 \times 6)/(12 + 6)]\ \Omega = 4\ \Omega.$$

The equivalent resistance R is given by

$$R = 2\ \Omega + 4\ \Omega + 1\ \Omega = 7\ \Omega.$$

$$I = \frac{\varepsilon}{R+r} = \frac{16\,V}{(7+1)\,\Omega} = 2\ \text{A}$$

Consider the resistors between A and B. If I_1 is the current in one of the 4 Ω resistors and I_2 the current in the other,

$$I_1 \times 4 = I_2 \times 4$$

$I_1 = I_2 = 1\text{A}$

Consider the resistances between C and D. If I_3 is the current in the 12 Ω resistor, and I_4 in the 6 Ω resistor.

$$I_3 \times 12 = I_4 \times 6, \text{ i.e., } I_4 = 2I_3$$
$$\text{But, } I_3 + I_4 = I = 2 \text{ A}$$
$$\text{Thus, } I_3 = \left(\frac{2}{3}\right) \text{A}, \; I_4 = \left(\frac{4}{3}\right) \text{A}$$

V_{AB} = I x 2= 2 x 2=4V, V_{BC}= 2 x 1= 2V,

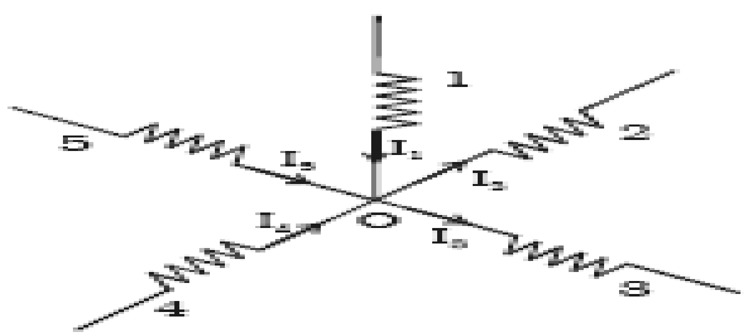
V_{CD}= I_3 x 12= ⅔ x 12= 8V

Kirchoff's laws

Ohm's law is applicable only for simple circuits. For complicatedcircuits, Kirchoff's laws can be used to find current or voltage. Thereare two generalisedlaws : (i) Kirchoff's current law (ii) Kirchoff's
voltage law

Kirchoff's first law (current law or junction rule)

Kirchoff's current law states that the algebraic sum of the currents meeting at any junction in a circuit is zero.



Sign convention

The convention is that, the current flowing towards a junction is positive and the current flowing away from the junction is negative.

$$I_1 + (-I_2) + (-I_3) + I_4 + I_5 = 0$$

$$I_1 + I_4 + I_5 = I_2 + I_3.$$

The sum of the currents entering the junction is equal to the sum of the currents leaving the junction.

This law is a consequence of **conservation of charges.**

Kirchoff's second law (voltage law or loop rule)

Kirchoff's voltage law states that the algebraic sum of the products of resistance and current in each part of any closed loop is equal to the algebraic sum of the emf's in that closed loop.

This law is a consequence of **conservation of energy.**

Sign Convention

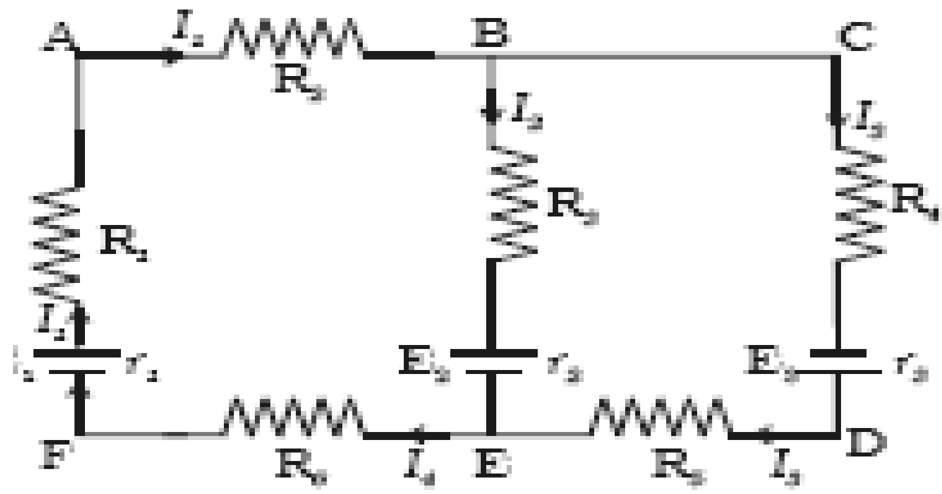
In applying Kirchoff's laws to electrical networks ,**the current is assumed to be positive, if the direction of current flow is clockwise in one convention** or the opposite also can be followed. If the assumed direction of current is not the actual direction, then on solving the problems, the current will be found to have **negative sign.**

If the result is positive, then the assumed direction is the same as actual direction.

It should be noted that, once the particular direction has been assumed, the same should be used throughout the problem.

Illustration

Let us consider the electric circuit given in Fig

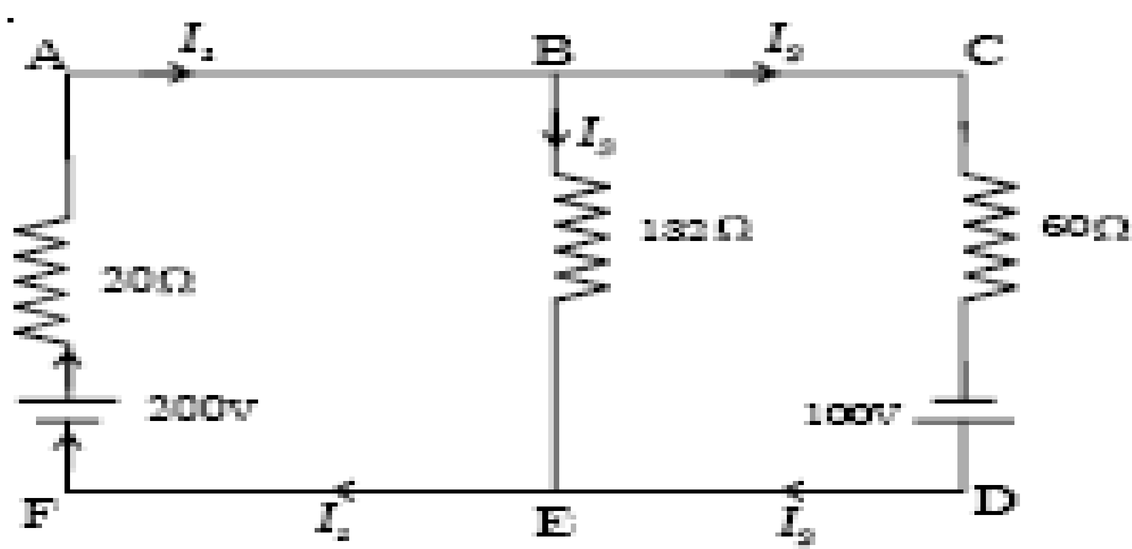


Considering the closed loop ABCDEFA,
$I_1R_2+I_3R_4+I_3r_3+I_3R_5+I_4R_6+I_1r_1+I_1R_1=E_1+E_3$

For the closed loop ABEFA
$I_1R_2+I_2R_3+I_2r_2+ I_2r_2+I_4R_6+I_1r_1+I_1R_1 = E_1-E_2$

Negative sign in E_2 indicates that it sends current in the anticlockwise direction.

Illustration I



Applying first law to the Junction B,

$$I_1 - I_2 - I_3 = 0$$

$$\therefore \quad I_3 = I_1 - I_2 \quad \ldots(1)$$

For the closed loop ABEFA,

$$182\ I_3 + 20I_1 = 200 \ \ldots(2)$$

Substituting equation (1) in equation (2)

$$182\ (I_1 - I_2) + 20I_1 = 200$$

$$152I_1 - 182I_2 = 200 \ \ldots(3)$$

For the closed loop BCDEB,

$$60I_2 - 182I_3 = 100$$

substituting for I_3,

$$\therefore \quad 60I_2 - 182\ (I_1 - I_2) = 100$$

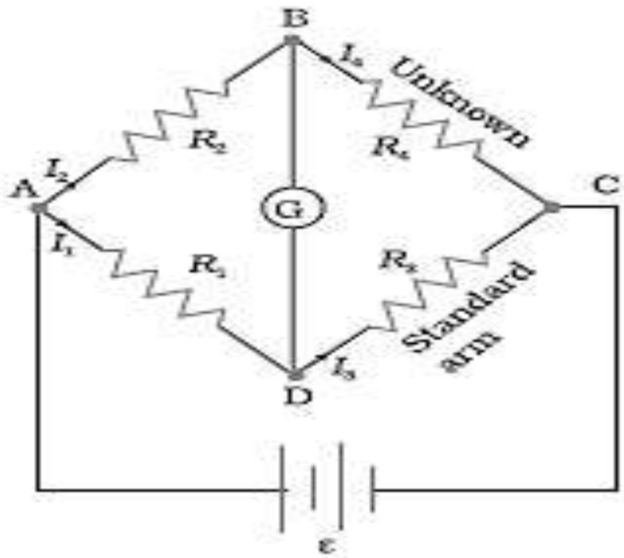
$$-182I_1 + 192I_2 = 100 \ \ldots(4)$$

Solving equations (3) and (4), we obtain

$$I_1 = 4.39\ A \text{ and } I_2 = 3.54\ A$$

WHEATSTONE BRIDGE

As an application of Kirchhoff's rules consider the circuit shown in figure, which is called the Wheatstone bridge.



The bridge has four resistors R_1, R_2, R_3 and R_4. Between points A and C a voltage source is connected. AC is called the battery arm. Between the other two vertices, B and D, a galvanometer G (which is a device to detect currents) is connected. This line, shown as BD in the figure, is called the galvanometer arm.
For simplicity, we assume that the cell has no internal resistance. In general there will be currents flowing across all the resistors as well as a current I_g through G.
In the case of a balanced bridge where the resistors are such that $I_g = 0$. We can easily get the balance condition, such that there is **no current through G**

In this case, the Kirchhoff's junction rule applied to junctions D and B immediately gives us the relations $\mathbf{I_1 = I_3}$ **and** $\mathbf{I_2 = I_4}$**.**

Next, we apply Kirchhoff's loop rule to closed loops ADBA and CBDC. The loop ABDA gives
–I1 R1 + 0 + I2 R2 = 0 (Ig = 0) …..1
and the loop CBDC gives,

I2 R4 + 0 – I1 R3 = 0 …..2

From equation 1 we obtain

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

From equation 2 we obtain

$$\frac{I_1}{I_2} = \frac{R_4}{R_3}$$

The above two equations give

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

This last equation relating the four resistors is called the balance condition for the galvanometer to give zero or null deflection.

The Wheatstone bridge and its balance condition provide a practical method for determination of an unknown resistance.

Suppose R_4 is an unknown resistance , at balance condition,

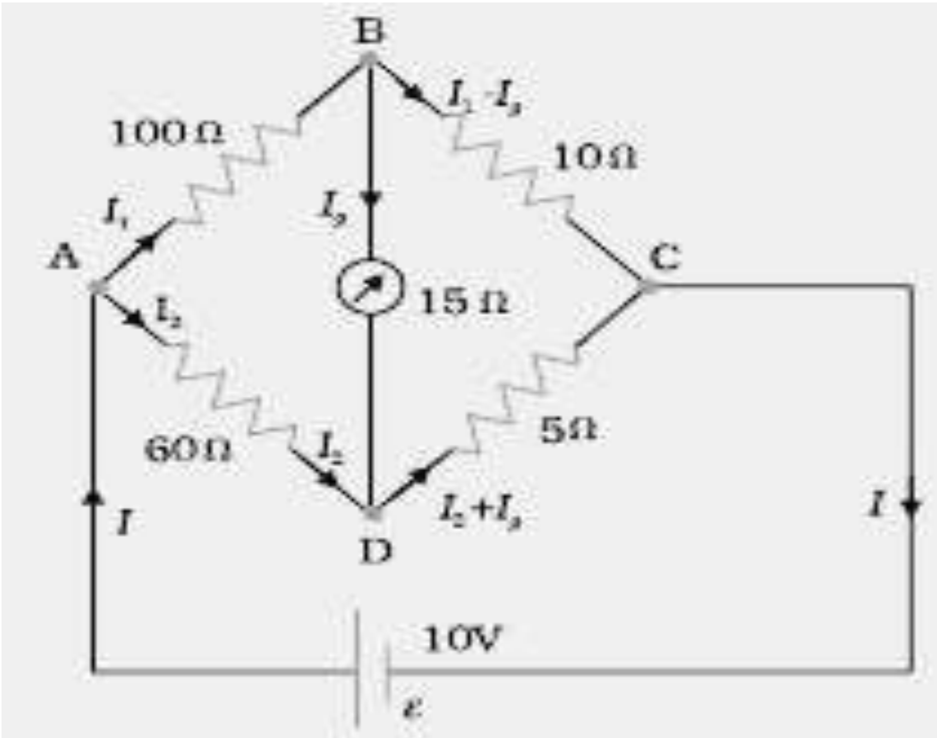
$$R_4 = R_3 \frac{R_2}{R_1}$$

Text book example:

The four arms of a Wheatstone bridge *have the following resistances:*

AB = 100Ω, BC = 10Ω, CD = 5Ω, and

DA = 60Ω.



Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.

Solution:

Considering the mesh BADB, we have

$100\ I_1 + 15\ I_g - 60\ I_2 = 0$

or $20\ I_1 + 3\ I_g - 12\ I_2 = 0$ ….. 1

Considering the mesh BCDB, we have

$10\ (I_1 - I_g) - 15I_g - 5\ (I_2 + I_g) = 0$

$10I_1 - 30I_g - 5I_2 = 0$

$2I_1 - 6I_g - I_2 = 0$ ….. 2

Considering the mesh ADCEA,

$60I_2 + 5\ (I_2 + I_g) = 10$

$65I_2 + 5I_g = 10$

$13I_2 + I_g = 2$ ….. 3

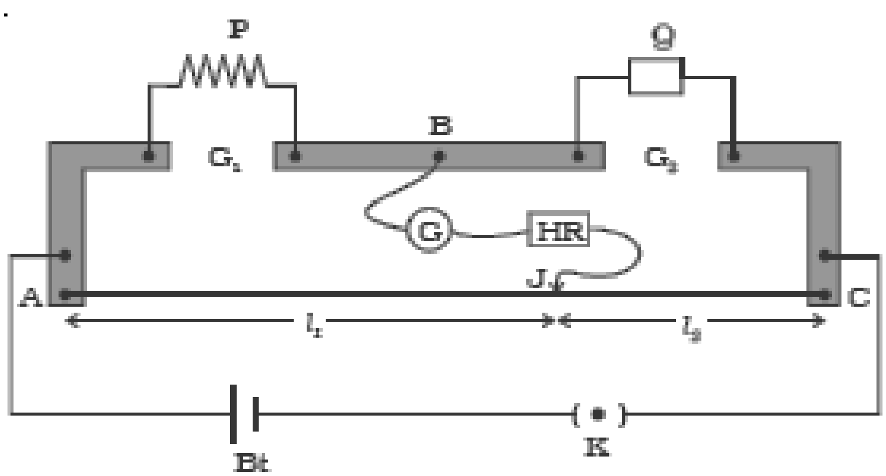
multiplying equation 2 by 10

$20I_1 - 60I_g - 10I_2 = 0$ …… 4

from equations 1 and 4

$63I_g - 2I_2 = 0$
$I_2 = 31.5I_g$
Substituting the value of I_2 into equation 3
we get
$13 (31.5I_g) + Ig = 2$
$410.5\ Ig = 2$
$Ig = 4.87$ mA.

Metrebridge
Metrebridgeis one form of W h e a t s t o ne 's bridge. It is as shown in the figure.



It consistsof thick strips of copper, of negligible resistance, fixed toa wooden board. There are two gapsG1 and G2 between These strips.
A uniform **manganin** wire AC of length one metre **whose temperature coefficient is low**, and **resistivity is high** is stretched along a metre scale and it sends are soldered to two **thick** copper strips.

An unknown resistance P is connected in the gap G1 and a standard resistance Q is connected in the gap G2.

A metal jockey J is connected to B through a galvanometer (G) and a high resistance (HR) and it can make contact at any point on the wire AC. Across the two ends of the wire, a Leclanche cell and a key are connected.

Adjust the position of metal jockey on metre bridge wire so that the galvanometer shows zero deflection. Let the point be J. The portions AJ and JC of the wire now replace the resistances R and S of

Wheatstone's bridge.Then

$$\frac{P}{Q} = \frac{R}{S} = \frac{r.AJ}{r.JC}$$

where r is the resistance per unit length of the wire.

$$\frac{P}{Q} = \frac{AJ}{JC} = \frac{l_1}{l_2}$$

where $AJ = l_1$ and $JC = l_2$

$$\therefore \quad P = Q\frac{l_1}{l_2}$$

The error in the value of unknown resistance can be eliminated, if another set of readings are taken with P and Q interchanged and the average value of P is found, provided the balance point J is near the midpoint of the wire AC.

Determination of specific resistance

The specific resistance of the material of a wire is determined by knowing the resistance (P), radius (r) and length (L) of the wire using the expression

$$\rho = \frac{P\pi r^2}{L}$$

Class assignment:

1. Why manganin is used as bridge wire instead of copper?

2. What is the advantage of high value of ρ and low value of α for the bridge wire?

3.Why the wire is connected by thick copper strips at the ends ?

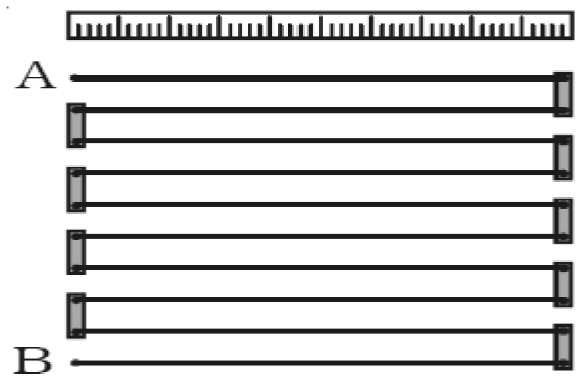
4.What is the advantage of connecting a high resistance in series with the galvanometer?

5. Why the balance point must be preferably near the midpoint of the wire AC?

Potentiometer

The Potentiometer isan instrument used forthe measurement of potential difference.

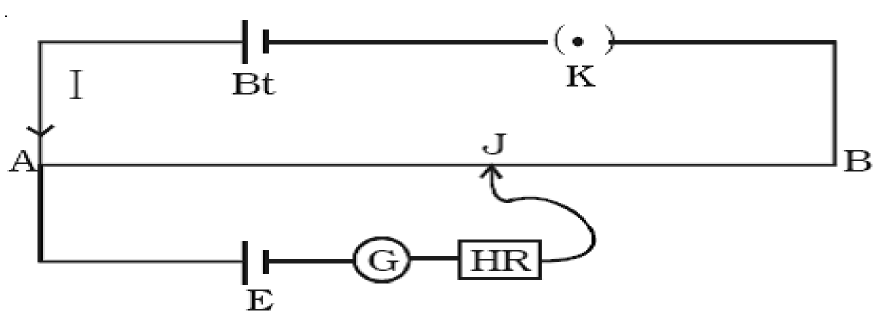
It consists of a ten metre long uniform wire ofmanganin or constantan stretched in ten segments, each of one metre length. The segments are stretched parallel to each other on a horizontal wooden board. The ends of the wire are fixed to copper strips with binding screws. A metre scale is fixed on the board ,parallel to the wire. Electrical contact with wires is established bypressing the jockey J.



Principle of potentiometer

A battery Bt is connected between the ends A and B of a potentiometer wire through a key K. A steady current I flows through the potentiometer. This forms the primary circuit.

A primary cell is connected in series with the positive terminal A of the potentiometer, a galvanometer, high resistance and jockey. This forms the secondary circuit.



If the potential difference between A and J is equal to the emf of the cell, no current flows through the galvanometer. It shows zero deflection. AJ is called the balancing length.

If the balancing length is *l*, **the potential difference across AJ = I*rl*** where ***r*** is the resistance per unit length of the potentiometer wire and **I** the current in the primary circuit.
p d across given length of a potentiometer wire is directly proportional to the balancing length when a steady current flows through it.

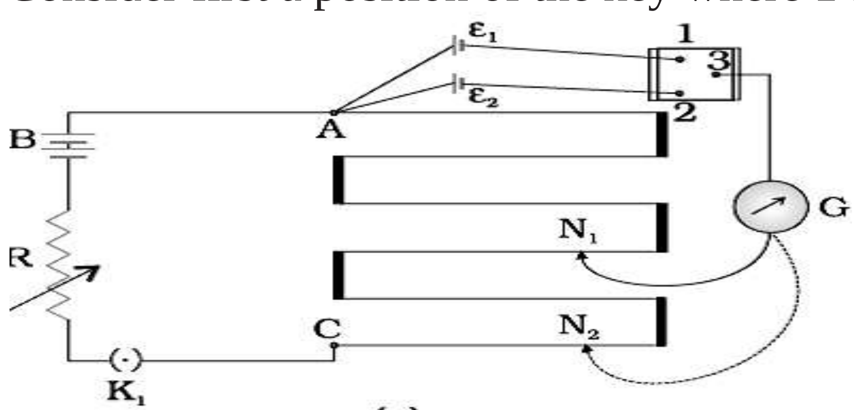
$\therefore \varepsilon = Irl$,

since I and *r* are constants, **$\varepsilon \propto l$ .Hence emf of the cell is directly proportional to its balancing length. This is the principle of a potentiometer.**

Comparison of emfs of two given cells using potentiometer.

The potentiometer wire ACis connected in series with a battery B Key (K_1), rheostat (R) . This forms the primary circuit. The end A of potentiometer is connected to the positive terminals of both the cells in the secondary circuit of emfs ε_1 and ε_2.The points marked 1, 2, 3 form a two way key.

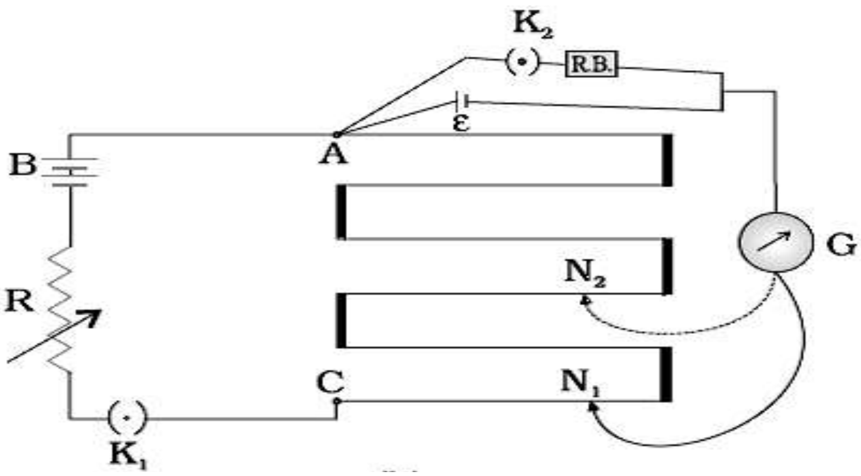
Consider first a position of the key where 1 and 3 are connected



so that the galvanometer is connected to ε1. The jockey is moved along the wire till at a point N1, at a distance l_1 from A, there is no deflection in the galvanometer.
The potential difference across the balancing length l_1 = Irl_l.
Then, by
the principle of potentiometer,
$\varepsilon_1 = Ir\, l_l$...(1)
Similarly, the position of the key where 2 and 3 are connected so that the galvanometer is connected to cell $\varepsilon_2$of emf ε_2 is balanced against l_2.The potential difference across the balancing length is
l_2 = Irl2, then
$\varepsilon_2 = Irl_2$...(2)
Dividing (1) by (2) we get

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

Determination of internal resistance of a cell using potentiometer.
We can also use a potentiometer to measure internal resistance of a cell . For this the cell of emf ε ,whose internal resistance *r* is to be determined is connected across a resistance box through a key K_2, asshown in the figure.



With key K_2 open, balance is obtained at length l_1 (AN1). Then, $\varepsilon = kl_1$ **where k =I r is known as potential gradient ,Pd per unit length,**

When key K_2 is closed, the cell sends a current I through the resistance box *R*. If *V* is the terminal potential difference of the cell and balance is obtained at length l_2 (AN2),

$V = kl_2$

So, we have

$$\varepsilon/V = l_1/l_2$$

But, $\varepsilon = I(r + R)$ and $V = IR$. This gives

$$\varepsilon/V = (r+R)/R$$

from the above two equations we get

$$(R+r)/R = l_1/l_2$$

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

Where r is the internal resistance of the cell and R is the external resistance.

Class assignment

1)Define potential gradient give its unit.

2) Which of the following a 4m length or 10m length potentiometer is more sensitive , when connected to the same voltage source? Justify

Heating effect : Joule's law

In a conductor, the free electrons are always at random motion making collisions with ions or atoms of the conductor.

When a voltage V is applied between the ends of the conductor, resulting in the flow of current I, the free electrons are accelerated. Hence the electrons gain energy at the rate of VI per second. This increase in energy is nothing but the thermal energy of the conductor.

Thus for a steady current I, the amount of heat produced in time t is

$H = V\,It$

For a resistance R,

$H = I^2Rt$ and

$$H = \frac{V^2}{R}\,t$$

The above relations were experimentally verified by Joule and are known as Joule's law of heating. It states that the **heat**

produced is (i) directly proportional to the square of the current for a given R and t
(ii) directly proportional to resistance R for a given I and t
(iii) directly proportional to the time of passage of current t for given I and R.

Uses of heating effect of electric current

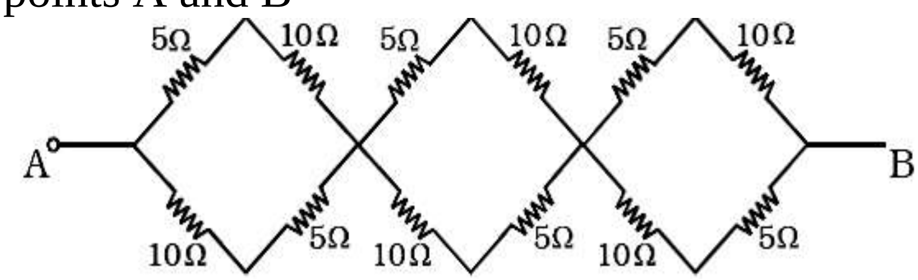
It is used in electrical appliances like i) electric iron, water heater , toaster to convert electric to heat energy.
ii) filament lamps to convert electric energy into heat and light energy.
iii) in safety fuse to melt and break the circuit in the event of short circuit and overload to prevent electrical fire.

Disadvantage of heating effect of electric current

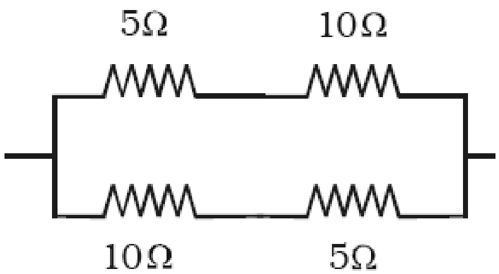
In some cases such as transformers and dynamos, Joule heating effect is undesirable. These devices are designed in such a way as to reduce the loss of energy due to heating.

Additional numerical problems with solution

1.In the given network, calculate the effective resistance between
points A and B



The network has three identical units. The simplified form of one unit is given as



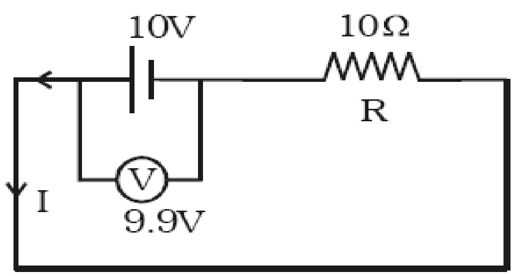
The equivalent resistance of one unit is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{15} + \frac{1}{15}$$

R_p = 7.5 Ω

Reffective is R_{eff}= 3 x R_p = 22.5Ω.

2. A 10 Ω resistance is connected in series with a cell of emf 10V. A voltmeter is connected in parallel to a cell, and it reads. 9.9 V . Find internal resistance of the cell.



R = 10 Ω ;E = 10 V ; V = 9.9 V ; r = ?

$$r = \left(\frac{E-V}{V}\right)R$$

$$= \left(\frac{10-9.9}{9.9}\right) \times 10$$

r = 0.101 Ω

Text book numerical problems

1.*The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery?*

2.*A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0 °C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70×10^{-4} $°C^{-1}$*

3. (a) In a metre bridge the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5 Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips?

(b) Determine the balance point of the bridge above if X and Y are interchanged.

(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?